Minimum Weight Design of an Orthogonally Stiffened Waffle Cylindrical Shell with Buckling Constraint

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The minimum weight design of an orthogonally stiffened waffle cylindrical shell with simple supported edges is studied. The minimum eigenvalues and the corresponding mode numbers for buckling are searched by the sectioning method or the gradient method. The formulations for the buckling analysis include two loading cases: uniform axial compressive stress; and bending, as well as uniform, axial stresses. The optimization method used is the steepest descent method with state inequality constraint. Six design variables are considered: thickness of the shell; depth and width of the longitudinal stiffeners; depth and width of the circumferential stiffeners; and spacing between the stiffeners. Three design examples are performed: two with uniform compression and one with bending load. When compared with three previously published designs, the reductions in weight are found to be 32.7, 60.2, and 54.7%, respectively. Such results suggest that tremendous weight reduction can be achieved by appropriate wall stiffening and that this method is highly effective for minimum weight design.

Nomenclature

a	= spacing between stiffeners
d_1, d_2	= depths of the longitudinal and circumferential stiffeners, respectively
ds	= step size in control space, in.
E	= modulus of elasticity
h	= thickness of the shell skin
L	= length of the cylindrical shell
m, n	= longitudinal half-wave and circumferential full-wave numbers, respectively
N_b	= maximum bending stress, lbs/in.
N_{bo}	= buckling value of N_b
N_c	= uniform axial compressive stress, lbs/in.
N_{co}	= buckling value of N_c
r	= mean radius of the cylindrical shell skin
t_l, t_2	= widths of the longitudinal and circumferential stiffeners, respectively
$\{u\}$	= vector of design parameters
U_{mn}, V_{mn}, W_{mn}	= displacement amplitudes in the x, θ, and z directions, respectively
W	= weight of the cylinder
x, θ, z	= cylindrical coordinates shown in Fig. 1
α, β	= Lagrange multipliers
λ	$= N_c(m\pi/L)^2$, eigenvalue
ν	= Poisson's ratio
$\{\ \}, \lfloor\ \rfloor, [\ ]$	= column, row, and the rectangular matrices, respectively.

Introduction

THE thin cylindrical shell is one of the most common structural components in the modern flight vehicles. It is known that it may be subjected to high axial compression and that its bending rigidity and buckling strength are weak. It is also known that the buckling strength for the cylindrical shell is notoriously sensitive to the initial imperfections in the skin.

The use of wall-stiffening system can, however, not only increase the buckling strength without increasing the weight of the cylinder, but also diminish the sensitivity of the buckling load caused by initial imperfections.

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The buckling of a simply-supported waffle cylinder with 45° oriented stiffeners and under uniform axial compression was studied both theoretically and experimentally by Meyer.[1] The structural synthesis of integrally stiffened cylinders subjected to axial load and internal or external pressure was studied by Kicher,[2] Schmit, Morrow, and Kicher,[3] and Pappas and Amba-Rao,[4] etc. Kicher[2] treated the problem by using the constrained gradient method. Schmit, Morrow, and Kicher[3] applied a Fiacco-McCormick-type penalty function formulation to transform the basic inequality constrained minimization problem into a sequence of unconstrained minimization problems. Pappas and Amba-Rao[4] used a direct search algorithm with an interior-exterior penalty function formulation and pointed out the importance of allowing the use of an arbitrary starting point. By showing that a minimum weight design is not unique, and considering that a structural element which is designed for simultaneous occurrence of all possible modes of failure is extremely sensitive to geometric imperfections, Simitses and Ungbhakorn[5] performed a minimum weight design of the fuselage-type stiffened circular cylindrical shells subject to uniform axial compression. A survey of the recent advances in this subject was also presented in Ref. 5.

Recently, Baig, and Yang[6] studied the buckling of the simply-supported orthogonally-stiffened waffle cylinders (Fig. 1) under both uniform axial compression and bending load. It was found for one example in both Refs. 1 and 6 that the wall-stiffening can increase the buckling strength by 32.8% when compared with a monocoque cylinder with the same weight. By assuming that either $t_l = t_2$ or $d_l = d_2$ (see Fig. 1 for

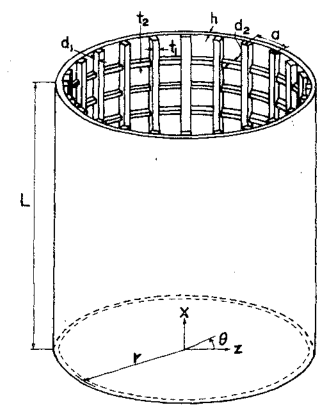

Fig. 1 An orthogonally stiffened waffle cylindrical shell.

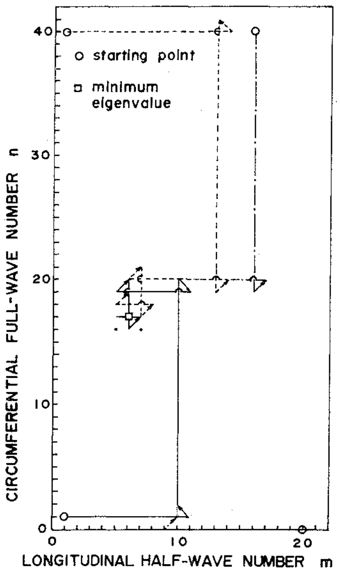

Fig. 2 Search for the lowest buckling load for a waffle cylinder ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $a = 3$ in.; $d_1 = d_2 = 0.2$ in.; $t_1 = t_2 = .125$ in.; $h = 0.05$ in.; $r = 48$ in.; $L = 50$ in.; weight $= 101.27$ lb; $N_{c0} = 789.82$ lb/in.).

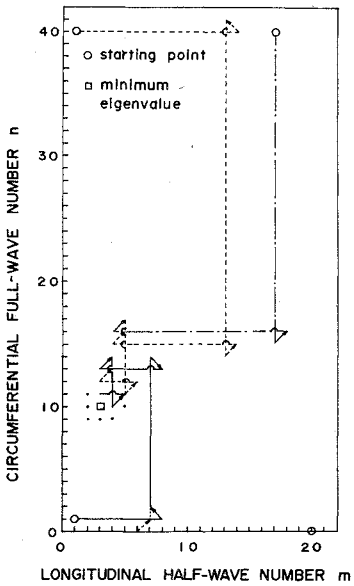

Fig. 3 Search for the lowest buckling load for a waffle cylinder ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $a = 3$ in.; $d_1 = d_2 = 0.37$ in.; $t_1 = t_2 = 0.148$ in.; $h = 0.03$ in.; $r = 48$ in.; $L = 50$ in.; weight $= 100.45$ lb; $N_{c0} = 1179.5$ lb/in.).

definitions), the number of parameters are reduced and a parametric study was performed in Ref. 6 to search for the parameters that yield the highest buckling strength for an example of a constant weight waffle cylinder. An increase in buckling strength by 98.2% in comparison with a monocoque cylinder of the same weight is noted. An example of a simply-supported waffle cylinder under pure bending moment was also analyzed in Ref. 6. The buckling strength was found to be increased by 40.36% as compared with a monocoque cylinder of the same weight.

For a stiffened cylindrical shell as shown in Fig. 1, there appears to be at least six parameters that can be varied to achieve a minimum weight design: h; a; t_1; t_2; d_1; and d_2. With six design variables in hand, a parametric study for finding the minimum weight appears not to be feasible. The rigorous optimization technique must be applied. Furthermore, some economic technique must be developed to search for the lowest eigenvalue (buckling load) once a set of design variables is specified in the optimization process.

In this paper, the "sectioning" method and the gradient method are used to search for the lowest eigenvalues. The method used in the optimization of the six design variables is the steepest descent method with buckling load as the state inequality constraint. The initial design does not have to be feasible. The eigenvalue equations given in Ref. 6 are reformulated to fit the optimization operation. Several design examples are performed.

Buckling Equations and Search Procedure

The eigenvalue equations for finding the buckling load for the simply supported orthogonally-stiffened waffle cylindrical shell subjected to uniform axial compression was derived in Ref. 6 as

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & (c_{22}-\lambda) & c_{23} \\ c_{31} & c_{32} & (c_{33}-\lambda) \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (1)$$

where the expressions c's are given in Ref. 6. To solve for the critical buckling load is to search for the smallest eigenvalue λ in the 3×3 matrix equations for various values of integers m and n. This problem may be expressed in the form that

$$N_{co} = \min_{(m,\,n)} \; [(L/m\pi)^2 \lambda(m,n)] \qquad (2)$$

with $m = 1, 2, \ldots$; and $n = 0, 1, 2, \ldots$. Equation (2) forms an unconstrained integer minimization problem.

Because of the possible existence of several local minima, Eq. (2) must be evaluated for every combination of m and n. A common engineering practice is to construct the so-called festoon curves for the buckling values for various numbers of m and n to find the minimum by inspection. When conducting the optimization analysis, the search for the minimum must be performed for each of the numerous sets of design variables. The search must be done by computer with efficient subroutines. In this paper, the sectioning approach as described in Ref. 7 is proposed.

The sectioning approach is evaluated by two examples. The first example is a simply-supported waffle cylinder described in the caption of Fig. 2. The search procedure is shown in Fig. 2 with four different starting points. The four paths all end at the same point with the minimum eigenvalue. The procedure can be described by, for example, the path that starts form $m = 1$ and $n = 1$. The number n is first held at one and m is increased. The path finds a local minimum at $m = 10$. The number m is thus brought back from 11 to 10 and held constant while n is increased. Antoher local minimum is found at $n = 10$ and m is then varied. The final minimum is found at $m = 6$ and $n = 17$ with a buckling load of 789.82 lbs/in. which agrees with that found in Ref. 6. The eigenvalue at this point is lower than those at the eight surrounding points.

The second example is a waffle cylinder with nearly the same weight as the first one but different dimensions (see caption of Fig. 3). The search procedure is shown in Fig. 3. It is noted that a local minimum is first found by all of the four paths at $m = 4$ and $n = 11$. The eigenvalues at (3, 11), (5, 11), (4, 10), and (4, 12) are all greater than this local minimum value. However, the absolute minimum is at $m = 3$ and $n = 10$ with $N_{c0} = 1179.5$ lbs/in. that agrees with that given in Ref. 6. All the eight points surrounding this point have larger eigenvalues. If Fig. 3 is plotted three dimensionally with the eigenvalue as the third dimension, the surface for eigenvalues will present an interesting inverted ridge shape near this minimum point and the nearest local minimum point.

In the subsequent minimum weight design, the previous technique will be used to search for the buckling load for each set of design parameters. To insure that the minimum thus found is a global minimum and not a local minimum, the eigenvalue is checked by considering every feasible value of m and n after the design parameters are finally converged through optimization.

When a simply-supported waffle cylinder is subjected to uniform axial compression and bending load, the axial stress is described by

$$N_x = N_b \cos\theta + N_c \qquad (3)$$

The buckling equations are given as[6]

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & (c_{33}-\lambda) \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ \psi \end{Bmatrix} = \begin{Bmatrix} F_{1_{mn}} \\ F_{2_{mn}} \\ F_{3_{mn}} \end{Bmatrix} \tag{4}$$

with

$$\psi = \frac{N_b}{2}\left(\frac{m\pi}{L}\right)^2 \{ (1+\delta_{1n}-\delta_{0n})\, W_{m(n-1)} + W_{m(n+1)} \} \tag{5}$$

where $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i\neq j$. Equation (4) forms an eigenvalue problem for N_b.

The order of the matrix in Eq. (4) is $3n$ by $3n$. For each of the numerous sets of design parameters, the eigenvalues must be solved for each value of m and the minimum eigenvalue is searched for. This requires tremendous computing effort. It is proposed to reduce the size of Eq. (4) by using the fact that the forces on the right of Eq. (4) are usually zeros.

Using the first two rows of Eq. (4), the amplitudes U_{mn} and V_{mn} may be written in terms of W_{mn} as

$$\begin{Bmatrix} U_{mn} \\ V_{mn} \end{Bmatrix} = -\frac{W_{mn}}{c_{11}c_{22}-c_{12}c_{21}} \begin{bmatrix} c_{22} & -c_{12} \\ -c_{21} & c_{11} \end{bmatrix} \begin{Bmatrix} c_{13} \\ c_{23} \end{Bmatrix} \tag{6}$$

Substituting Eq. (6) into the third row of Eq. (4) yields

$$-\frac{N_b}{2}\left(\frac{m\pi}{L}\right)^2 (1+\delta_{1n}-\delta_{0n})\, W_{m(n-1)} + \{ (c_{33}-\lambda) - (c_{31}c_{22}c_{13}-c_{31}c_{12}c_{23}-c_{32}c_{21}c_{13}+c_{32}c_{11}c_{23}) / (c_{11}c_{22}-c_{12}c_{21}) \} W_{mn} - \frac{N_b}{2}\left(\frac{m\pi}{L}\right)^2 W_{n(n+1)} = 0 \tag{7}$$

Equation (7) may be expanded to a tri-diagonal matrix form as

where

$$a_{mn} = \frac{1}{2}\left[\frac{m\pi}{L}\right]^2 \Big/ \left\{ (c_{33}-\lambda) - \frac{c_{31}c_{22}c_{13}-c_{31}c_{12}c_{23}-c_{32}c_{21}c_{13}+c_{32}c_{11}c_{23}}{c_{11}c_{22}-c_{12}c_{21}} \right\}$$

$\mu = 1/N_b$ (eigenvalue)

Such kind of eigenvalue problems is solved by the subroutines FIGI, RATQR, TINVIT, BAKVEC in EISPACK, from the Argonne National Laboratory.

The buckling value of the maximum bending stress N_{bo} is the smallest value of N_b (or largest value of μ) which is dependent upon the longitudinal half wave number m

$$N_{b0} = \underset{(m)}{\text{minimum}} \{ N_b(m) \} \tag{10}$$

Assuming that $N_b(m)$ is a unimodal function, a gradient method is applied for the minimization of the function N_b. Since m is an integer, the step size Δm is taken as unity. With the use of this method and with the reduced size of [Eq. (9)], computing cost is tremendously reduced. This method is used for each set of design parameters during optimization process. The buckling load found for the final set of parameters is, however, checked by considering every value of m to ensure that it is not just a local minimum.

Optimization Technique

The vector for the six design variables is defined as

$$\lfloor u \rfloor = \lfloor a\ d_1 d_2 t_1 t_2 h \rfloor \tag{11}$$

The weight W and the buckling stress N_0 for the orthogonally-stiffened cylindrical shell are functions of the design variables, i.e.

$$W = W(\langle u \rangle) \tag{12}$$

$$\begin{bmatrix} \mu & -a_{mo} & & & & \\ -2a_{m1} & \mu & -a_{m1} & & & \\ & -a_{m2} & \mu & -a_{m2} & & \\ & & & \ddots & & \\ & & & & -a_{m\ell} & \mu \end{bmatrix} \begin{Bmatrix} W_{mo} \\ W_{m1} \\ W_{m2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{m\ell} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Bmatrix} \tag{8}$$

or

$$\begin{bmatrix} 0 & a_{mo} & & & & \\ 2a_{m1} & 0 & a_{m1} & & & \\ & a_{m2} & 0 & a_{m2} & & \\ & & & \ddots & & \\ & & & & a_{m\ell} & 0 \end{bmatrix} \begin{Bmatrix} W_{mo} \\ W_{m1} \\ W_{m2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{m\ell} \end{Bmatrix} = \mu \begin{Bmatrix} W_{mo} \\ W_{m1} \\ W_{m2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_{m\ell} \end{Bmatrix} \tag{9}$$

$$N_0 = N_0(\langle u \rangle) \tag{13}$$

The optimization problem may be expressed as

$$\text{To minimize } W = W(\langle u \rangle) \tag{14}$$

$$\text{Subjected to } \phi(\langle u \rangle) = N_0(\langle u \rangle) - C_0 \geq 0 \tag{15}$$

where C_0 is the maximum allowable buckling stress. If the design variables $\langle u \rangle$ reach the inequality constraint boundary defined in Eq. (15), the constraint condition $\phi(\langle u \rangle) = 0$ is then applied.

The necessary equations for optimization are developed by means of the steepest descent method with state variable and control variable inequality constraint.[8]

The problem on the constraint boundary may be rewritten as follows:

$$\text{To minimize} \quad W = W(\langle u \rangle) \tag{16}$$

$$\text{Subject to} \quad \phi(\langle u \rangle) = 0 \tag{17}$$

The differential of weight is given by

$$\mathrm{d}W = \left\lfloor \frac{\partial W}{\partial u} \right\rfloor \{\mathrm{d}\} \tag{18}$$

where

$$\left\lfloor \frac{\partial W}{\partial u} \right\rfloor = \left\lfloor \frac{\partial W}{\partial a} \quad \frac{\partial W}{\partial d_1} \quad \frac{\partial W}{\partial d_2} \quad \frac{\partial W}{\partial t_1} \quad \frac{\partial W}{\partial t_2} \quad \frac{\partial W}{\partial h} \right\rfloor \tag{19}$$

$$\mathrm{d}W = \left\lfloor \frac{\partial W}{\partial u} \right\rfloor \{\mathrm{d}u\} \tag{20}$$

where

$$\left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor = \left\lfloor \frac{\partial \phi}{\partial a} \quad \frac{\partial \phi}{\partial d_1} \quad \frac{\partial \phi}{\partial d_2} \quad \frac{\partial \phi}{\partial t_1} \quad \frac{\partial \phi}{\partial t_2} \quad \frac{\partial \phi}{\partial h} \right\rfloor \tag{21}$$

A step size in control space is introduced as

$$\mathrm{d}s^2 = \lfloor \mathrm{d}u \rfloor [T] \{\mathrm{d}u\} \tag{22}$$

where $[T]$ is a positive definite symmetric matrix. Adjoining Eqs. (20) and (22) to Eq. (18), it is obtained that

$$\mathrm{d}W = \left\lfloor \frac{\partial W}{\partial u} \right\rfloor \{\mathrm{d}u\} + \alpha\left(\mathrm{d}\phi - \left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor \{\mathrm{d}u\}\right) + \beta\left(\mathrm{d}s^2 - \lfloor \mathrm{d}u \rfloor [T] \{\mathrm{d}u\}\right) \tag{23}$$

where α and β are the Lagrange multipliers.

To minimize $\mathrm{d}W$ is to vanish $\mathrm{d}^2 W$. Thus

$$\left(\left\lfloor \frac{\partial w}{\partial u} \right\rfloor - \alpha \left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor - 2\beta \lfloor \mathrm{d}u \rfloor [T]\right)\{\mathrm{d}^2 u\} = 0 \tag{24}$$

Solving Eq. (24) for $\{\mathrm{d}u\}$ gives

$$\{\mathrm{d}u\} = [T]^{-1}\left(\left\{\frac{\partial W}{\partial u}\right\} - \alpha\left\{\frac{\partial \phi}{\partial u}\right\}\right)/2\beta \tag{25}$$

The step size is obtained by substituting Eq. (25) into Eq. (22)

Toward this end, the following scalars are defined

$$I_{\phi w} = \left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor [T]^{-1}\left\{\frac{\partial W}{\partial u}\right\}$$

$$I_{\phi\phi} = \left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor [T]^{-1}\left\{\frac{\partial \phi}{\partial u}\right\}$$

$$I_{ww} = \left\lfloor \frac{\partial W}{\partial u} \right\rfloor [T]^{-1}\left\{\frac{\partial W}{\partial u}\right\} \tag{28}$$

Based on these definitions, Eq. (27) is abbreviated as

$$\alpha = (I_{\phi w} - 2\beta\, \mathrm{d}\phi)/I_{\phi\phi} \tag{29}$$

Substituting Eq. (29) into Eq. (26) results in a lengthy quadratic equation in 2β. The solution for this equation is

$$2\beta = \pm\left(\frac{I_{ww} - I_{\phi w}^2/I_{\phi\phi}}{\mathrm{d}s^2 - \mathrm{d}\phi^2/I_{\phi\phi}}\right)^{1/2} \tag{30}$$

To minimize $\mathrm{d}W$ requires the second derivative of $\mathrm{d}W$ to be positive. Since

$$\mathrm{d}^3 W = -2\beta \lfloor \mathrm{d}^2 u \rfloor [T] \{\mathrm{d}^2 u\} \tag{31}$$

and matrix $[T]$ is positive definite, the multiplier β must be negative. Equation (30) thus becomes

$$2\beta = -\left(\frac{I_{ww} - I_{\phi}^{2}{}_{w}/I_{\phi\phi}}{\mathrm{d}s^2 - \mathrm{d}\phi^2/I_{\phi\phi}}\right)^{1/2} \tag{32}$$

With the two Lagrange multipliers α and β obtained in Eqs. (29) and (32), the incremental vector of the design variables defined in Eq. (25) is finally obtained

$$\{\mathrm{d}u\} = -\left(\frac{\mathrm{d}s^2 - \mathrm{d}\phi^2/I_{\phi\phi}}{I_{ww} - I_{\phi w}^2/I_{\phi\phi}}\right)^{1/2} [T]^{-1} \left(\left\{\frac{\partial w}{\partial u}\right\} - \frac{I_{\phi w}}{I_{\phi\phi}}\left\{\frac{\partial \phi}{\partial u}\right\}\right) + [T]^{-1}\left\{\frac{\partial \phi}{\partial u}\right\}\left(\frac{\mathrm{d}\phi}{I_{\phi\phi}}\right) \tag{33}$$

where $\mathrm{d}\phi$ is given by the amount that violates Eq. (17), or

$$\mathrm{d}\phi = -k\phi(\{u\}) \text{ with } 0 < k \leq 1 \tag{34}$$

In all cases the constant k in Eq. (34) is chosen to be unity. If, with this choice, the quantity within the square root in Eq. (33) becomes negative, Eq. (33) will be replaced by that

$$\{\mathrm{d}u\} = [T]^{-1}\left\{\frac{\partial \phi}{\partial u}\right\}\left(\frac{\mathrm{d}\phi}{I_{\phi\phi}}\right) \tag{35}$$

The derivatives of weight with respect to design variables $\{\partial W/\partial u\}$ are calculated numerically. The derivatives of the constraint with respect to design variables $\{\partial \phi/\partial u\}$ are computed via the derivatives of the eigenvalues with respect to the design variables $\{\partial \lambda/\partial u\}$ which are derived as follows.[9]

A standard eigenvalue problem is in the form that

$$[[k] - \lambda[I]]\{q\} = 0 \tag{36}$$

$$\mathrm{d}s^2 = \left(\left\lfloor \frac{\partial W}{\partial u} \right\rfloor - \alpha \left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor\right) [T]^{-1} \left(\left\{\frac{\partial W}{\partial u}\right\} - \alpha\left\{\frac{\partial W}{\partial u}\right\}\right)/4\beta^2 \tag{26}$$

The Lagrange multiplier α is obtained by substituting Eq. (25) into Eq. (20) and solving the resulting equation

$$\alpha = \left(\left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor [T]^{-1}\left\{\frac{\partial W}{\partial u}\right\} - 2\beta\, \mathrm{d}\phi\right)\left(\left\lfloor \frac{\partial \phi}{\partial u} \right\rfloor [T]^{-1}\left\{\frac{\partial \phi}{\partial u}\right\}\right)^{-1} \tag{27}$$

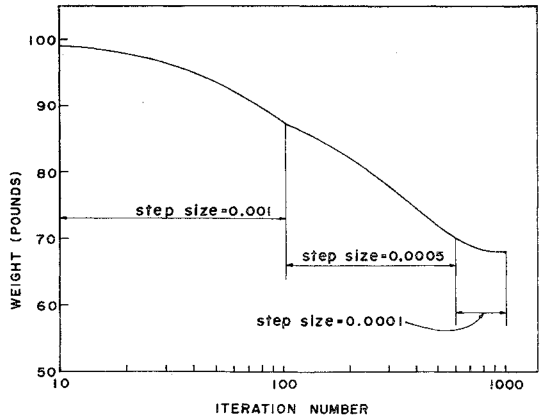

Fig. 4 Weight minimization of waffle cylinder with uniform axial buckling stress of 1179.51 lb/in. as the constraint.

where λ_i is the ith eigenvalue and $\{q_i\}$ is the corresponding eigenvector.

Premultiplying Eq. (36) by $\{q_i\}$ yields

$$\lfloor q_i \rfloor \left[[k] - \lambda_i [I] \right] \{q_i\} = 0 \tag{37}$$

Differentiating Eq. (37) with respect to one of the six parameters u_j gives

$$\lfloor q_i \rfloor \left[\frac{\partial [k]}{\partial u_j} - \frac{\partial \lambda_i}{\partial u_j} [I] \right] \{q_i\} = 0 \tag{38}$$

Using the fact that $\lfloor q_i \rfloor \{q_i\} = 1$, the derivative of the eigenvalue is finally obtained from Eq. (38).

$$\frac{\partial \lambda_i}{\partial u_j} = \lfloor q_i \rfloor \frac{\partial [k]}{\partial u_j} \{q_i\} \tag{39}$$

Results for Design Examples

A. Waffle Cylinder Under Uniform Axial Compression

In Ref. 6, a simply-supported orthogonally stiffened waffle cylinder (as described in the caption of Fig. 2) was found to have a buckling load of 789.82 lb/in. which is 32.8% higher than that (594.84 lb/in.) of a monocoque cylinder of the same weight. The weight of the cylinder was then held constant and a parametric study was made to find the highest buckling strength. The best design found in Ref. 6 (as described in the caption of Fig. 3) gives a buckling load of 1179.5 lb/in. (49.4% gain).

As a first example in the present minimum weight analysis, the design variables that give the highest buckling load in Ref. 6 are chosen as the initial design variables. The constraint buckling load is held at 1179.51 lb/in. The results are shown in Fig. 4. Three different kinds of step size are chosen. It is seen that the smaller the step size, the lighter weight and the more accurate constraint are obtained. The step size cannot, however, be decreased drastically because of the increase in computing cost and the loss of accuracy by numerical differentiation and other numerical computation. The minimum weight achieved in the design is 67.639 lb which is 32.7% lighter than that of the design given in Ref. 6. It must be noted that such improvement can also be achieved by the parametric search technique as used in Ref. 6 if the region of search is expanded in the same direction (increasing d/t values).

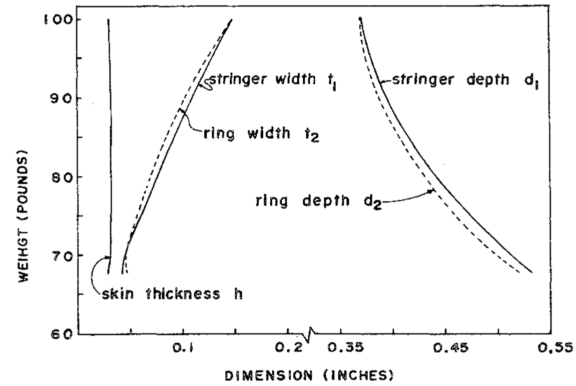

Fig. 5 Variation of design variables during the optimization shown in Fig. 4.

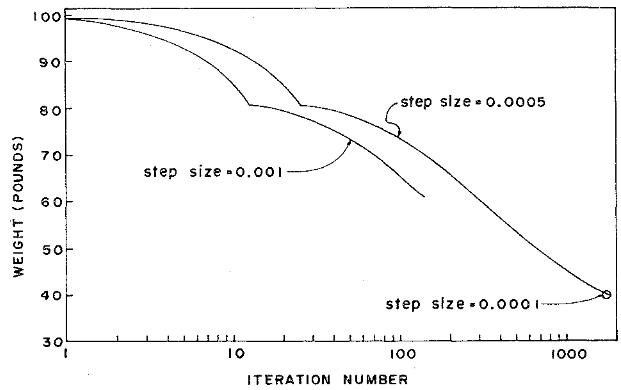

Fig. 6 Weight minimization of waffle cylinder with uniform axial buckling stress of 1179.5 as the initial design value and 789.82 lb/in. as the constraint.

For clarity of presentation, selected numerical results are given in Table 1. The change of design variables are shown in Fig. 5. Since the stiffener spacing remains almost unchanged, it is not shown in the figure. As the optimization proceeds, the skin thickness reduces slightly, the stiffener widths reduce significantly, and the stiffener depths increase pronouncedly. These variations appear to be reasonable since such changes will increase the cross sectional moments of inertia for the stiffeners yet without gaining weight. The increase in buckling load is obviously attributed to such increases in the moments of inertia.

The second example is to minimize the weight of the waffle cylinder with a lower buckling constraint $C_0 = 789.82$ lb/in. The initial design variables are the same as those chosen for the first example. The results are shown in Fig. 6. Three different kinds of step size are chosen. The smaller step size yields the lighter design. Since the initial design has higher buckling load than the buckling constraint, the weight reduces rapidly during the optimization until the buckling constraint

Table 1 Selected data for the results shown in Fig. 4 ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $r = 48$ in; $L = 50$ in.)

No. of steps	Step size in.	a in.	d_1 in.	d_2 in.	t_1 in.	t_2 in.	h in.	N_{co} lb/in.	Mode m,n	Weight lb.
0		3.0000	0.37000	0.37000	0.14800	0.14800	0.03000	1179.5	3,10	100.45
102	0.001	2.9953	0.40692	0.39634	0.09890	0.08975	0.03194	1179.5	4,11	87.062
310	0.0005	2.9916	0.45475	0.44209	0.06843	0.06288	0.03138	1179.1	3,10	77.797
1102	0.0001	2.9839	0.53205	0.51943	0.04155	0.04676	0.02884	1179.5	3,10	67.639

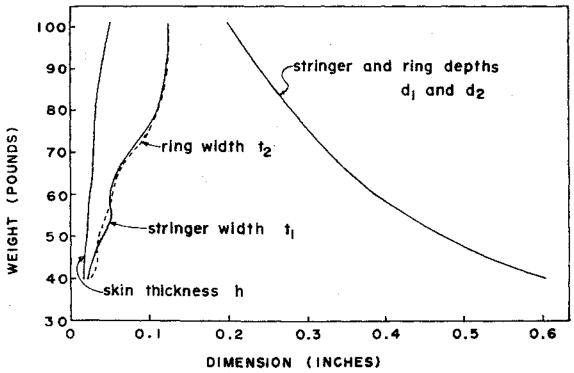

Fig. 7 Variation of design variables during the optimization with uniform axial buckling stress of 789.82 lb/in. as both the intial design value and the constraint.

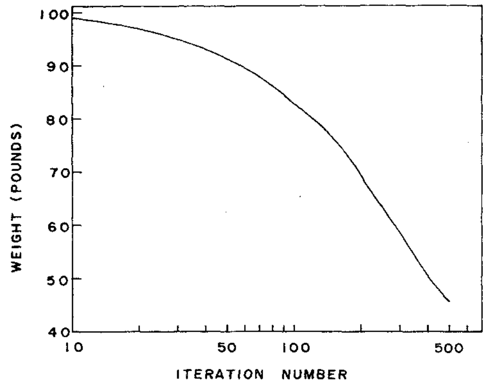

Fig. 8 Weight minimization of waffle cylinder with buckling maximum bending stress of 847.16 lb/in. as constraint.

is reached. On the constraint boundary, the weight reduces slowly. Considerable weight reduction can, however, be achieved. The minimum weight is found to be 39.969 lb which is 60.2% lighter than that of the initial design. Selected numerical results are presented in Table 2.

The initial design for this example was then changed to that as described in Table 3. This design has an initial buckling load of 789.82 lb/in which is of the same value as the buckling constraint. The final design is shown in Table 3. The variations of the design variables through the optimization process are shown in Fig. 7. Since the stiffener spacing changes only by 4.1%, its variation is not shown in Fig. 7. The difference between the variations in depth of the longitudinal and circumferential stiffeners is too small to be shown in the figure. As the optimization proceeds, the skin thickness reduces slightly, the stiffener widths reduce significantly and the stiffener depths increase pronouncedly.

For both examples, it is found that the buckling mode remains almost the same as that of the intial design during the optimization process. When the optimization is completed, the buckling load for the mode with m = 3 and n = 10 becomes almost identical to that for the mode m = 4 and n = 11.

B. Waffle Cylinder Under Pure Bending Load

An example of simply-supported waffle cylinder subjected to the distributed axial stress (bending) in the form of $N_b \cos\theta$ (lb/in.) was analyzed in Ref. 6. The example showed that wall-stiffening increases the buckling value of the maximum bending stress by 40.36% as compared with a monocoque cylinder of the same weight. The design parameters in that example are used here as the first set of initial design (see Table 4). The circumferential modes are represented by the geometric series of 28-terms. The results for weight minimization are shown in Fig. 8. The final design is given in Table 4. A reduction of weight by 54.7% is indicated. The second set of initial design parameters is also given in Table 4. This design has an initial buckling value of the maximum bending stress of 1336.4 lb/in. The final design is seen in Table 4 to be nearly the same as that obtained in the first case.

The variations of the first set of design parameters during the optimization are shown in Fig. 9. The stiffener spacing

Table 2 Selected data for the results shown in Fig. 6 ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $r = 48$ in; $L = 50$ in.)

No. of steps	Step size in.	a in.	d_1 in.	d_2 in.	t_1 in.	t_2 in.	h in.	N_{co} lb/in.	Mode m,n	Weight lb.
0		3.0000	0.37000	0.37000	0.14800	0.14800	0.3000	1179.5	3,10	100.45
138	0.001	2.9957	0.39623	0.39097	0.07339	0.06433	0.02190	789.77	4,11	60.95
0		3,0000	0.37000	0.37000	0.14800	0.14800	0.03000	1179.5	3,10	100.45
1688	0.0005	2.9773	0.60128	0.60546	0.02110	0.02491	0.01696	789.21	3,10	40.08
1756	0.0001	2.9771	0.60310	0.60763	0.02092	0.02472	0.01693	789.82	3,10	39.969

Table 3 Selected data for the results shown in Fig. 7 ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $r = 58$ in.; $L = 50$ in.)

No. of steps	Step size in.	a in.	d_1 in.	d_2 in.	t_1 in.	t_2 in.	h in.	N_{co} lb/in.	Mode m,n	Weight lb.
0		3.000	0.20000	0.20000	0.12500	0.12500	0.0500	789.82	6,17	101.27
1900	0.0005	2.8762	0.59538	0.60679	0.02110	0.02408	0.01693	789.43	3,9	40.342
2599	0.0001	2.8745	0.60600	0.61920	0.01983	0.02236	0.01692	789.48	4,10	39.643

Table 4 Selected data for the case of pure bending ($E = 1.06 \times 10^7$ psi; $\nu = 1/3$; $r = 48$ in.; $L = 50$ in.)

No. of steps	Step size in.	a in.	d_1 in.	d_2 in.	t_1 in.	t_2 in.	h in.	N_{bo} lb/in.	Mode	Weight lb.
0		3.0000	0.20000	0.20000	0.12500	0.12500	0.05000	847.16	6	101.27
500	0.001	2.9821	0.49922	0.49360	0.03995	0.03468	0.01764	847.13	4	45.83
0		3.0000	0.37000	0.37000	0.14800	0.14800	0.03000	1336.4	4	100.45
700	0.0005	2.9911	0.50819	0.47839	0.03741	0.03716	0.01791	847.05	4	46.021

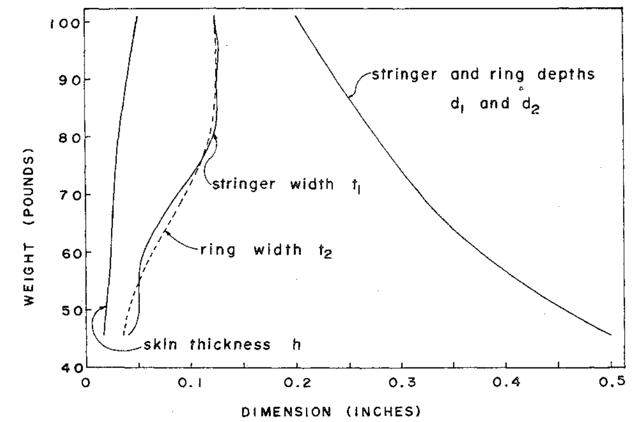

Fig. 9 Variation of design variables during the optimization shown in Fig. 8.

changes only by 0.7%, so this variation is not shown in Fig. 9. As the optimization proceeds, the skin thickness reduces somewhat, the stiffener widths reduce significantly, and the stiffener depths increase pronouncedly. The difference between the variations in depth of the longitudinal and circumferential stiffeners is too small to be shown in the figure.

Figure 10 shows the variation of buckling value of the maximum bending stress with number of longitudinal half wave for both the initial and final designs for the first case as described in Table 4. The curve after the optimization is steeper than the intial one. After the optimization, the buckling stresses for the cases $m=3$ and $m=4$ are almost identical.

The results for the normalized radial deflections ($\Sigma W_m^2=1$) along the circumferential direction are shown in Fig. 11 for three cases: the monocoque cylinder; the waffle cylinder; and the optimized waffle cylinder. Because of the symmetrical deformation, only half of the cylinder need be shown. It is interesting to note that the buckling mode shape tends to spread wide in the circumferential direction during the optimization. The spreaded-out mode can obviously take higher buckling load than the localized mode.

A CDC 6500 computer was used in performing the above calculations. The central processing time in the case of uniform axial load was approximately 0.07 sec for each iteration. The total central processing time for calculating the results given in Table 1 was 1.3 min. For computing the two cases in Table 2, the central processing time was 10 sec and 2.0 min, respectively. For computing the results of Table 3, the time was 3.02 min. In the case of pure bending, the central processing time was approximately 1.23 sec for each iteration. The computation of the two cases in Table 4 took 10.3 and 16.3 min, respectively.

Conclusions

A steepest descent optimization method has been used to formulate the buckling problem of simply-supported waffle cylinders. The uniform axial compression and the bending load are considered. Interesting conclusions can be drawn from the results obtained. The observation on the change of design variables (Figs. 5, 7, and 9) during the optimization shows that the differences between the variations of the stiffener depths d_1 and d_2 are very little. The results in all tables show that the spacing between stiffeners change very slightly during the optimization. It appears that the stiffener spacing may not have to be chosen as a design parameter and that imposing $d=d_1=d_2$ may not become a severe constraint. Consequently, the number of design variables may be reduced from the present six to four.

It appears that during the optimization the buckling strength are mostly gained by the increase in bending rigidities of the stiffeners. The step size should be small enough to maintain the buckling constraint. But smaller step results in

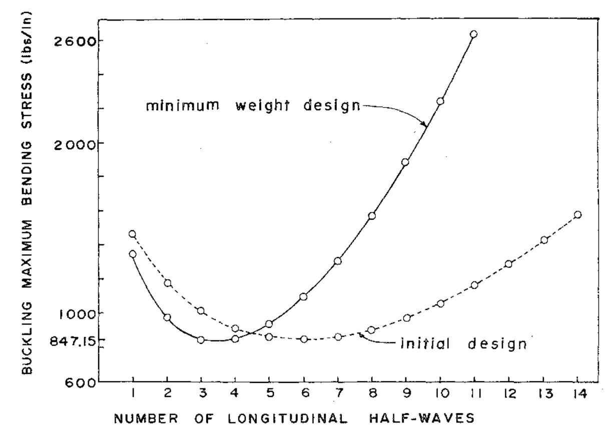

Fig. 10 Variation of buckling maximum bending stress with number of longitudinal half waves.

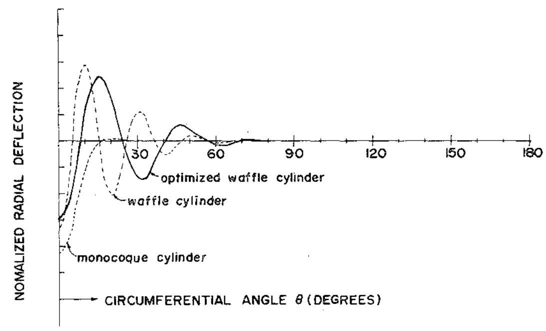

Fig. 11 The circumferential buckling mode for three cases: a) monocoque cylinder, $m=16$, $W=101.32$ lb, $N_{b0}=604.19$ lb/in.; b) waffle cylinder, $m=6$, $W=101.27$ lb, $N_{b0}=847.19$ lb/in; c) optimized waffle cylinder, $m=4$, $W=45.83$ lb, $N_{b0}=847.13$ lb/in.

increase in computer cost. It is suggested to use smaller step size only when the design approaches optimum.

There are, however, no design variable constraints imposed in this study. In practice, such constraints do exist due to fabricational inaccuracy and manufacturing difficulty. There should also be local buckling constraint of stiffeners and stress limit constraint. These constraints can be imposed without difficulty since the optimization method is already established here.

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